

Tensor Products and Kirchberg's Conjectures

Seminar¹ – Summer Semester 2023

J.Prof. Dr. Kristin Courtney and AR Dr. Sam Evington

Talks are assigned on a first-come-first-serve basis. Contact one of the organizers in person or by email to request a particular talk. (Later dates are subject to possible change depending on speaker schedules.)

You should arrange a meeting with one or both of the organizers to discuss a general outline as well as the treatment of supplementary/peripheral material before you begin preparing your lecture. A draft of the lecture notes should be submitted at least two weeks before the presentation, at which point a meeting with one or both of the organizers should be arranged to discuss the presentation and clarify any details from the material as needed. Written lecture notes should accompany any presentation that is to be graded.

Please notify us in advance if you wish to receive a grade for the presentation.

A summary of the upshots of lectures 1-9 will be regularly updated here.

1. Fundamentals for tensor products of C^* -algebras.

Speaker: Kristin Courtney

Date: 04.05.23

Contents: Important facts from tensor products of complex algebras and Hilbert space operators, highlights from maximal and spatial tensor products, tensor products and cp maps, injectivity and Arveson's theorem, operator systems.

Literature: [5, §3.1-3.3].

2. (Weak) Injectivity.

Speaker: Lukas Obermeyer

Date: 12.04.23

Contents: Lance's Weak Expectation Property (WEP), inclusions of tensor products, and max tensor of a nuclear map with identity factors through min tensor product.

Sources: [5, §3.6].

3. Exact Sequences.

Speaker: Vishnu Sankarankutty

Date: 19.04.23

Contents: Exactness of the functors $(\cdot) \otimes_{\min} A$ and $(\cdot) \otimes_{\max} A$ for a fixed C^* -algebra A , characterize nuclearity of maps via tensor products with $C^*(\mathbb{F})$.

Sources:[5, §3.7 up to pg. 96 (possibly omitting pg. 94), Corollary 3.8.8.]

4. Unique C^* -tensor norm on $B(\mathcal{H}) \otimes C^*(\mathbb{F})$ and consequences

Speaker: Sam Evington

Date: 26.04.23

Contents: Unique C^* -tensor norm on $B(\mathcal{H}) \otimes C^*(\mathbb{F})$ (via Pisier's free product strengthening), tensorial characterizations of LLP and WEP via $B(\mathcal{H})$ and $C^*(\mathbb{F})$, and nuclear embeddability \iff exactness.

Sources: [18, Proposition 8.8, Chapter 13], [19, Chapter 9], [5, §13.2].

5. Kirchberg's Local Lifting Property.

Speaker: Kristin Courtney

Date: 03.05.23

Contents: (Local) lifting property (LLP), LLP for nuclear C^* -algebras, LLP for $C^*(\mathbb{F}_n)$ (using Kasparov's Stinespring Dilation theorem and the noncommutative Tietze Extension theorem), Choi-Effros

¹The seminar is aimed at masters students who have seen Operator Algebras I. It can be used to fulfil a Specialization Supplement and Research Skills module (Ma-E) or a Specialization Module (Ma-S2 (Geometric Structures) or Ma-S3 (Operator Algebras and Noncommutative Geometry)).

and Effros-Haagerup lifting theorems.

Sources: [5, §13.1, Appendix C], [15, §3], [14, §2].

6. Connes Embedding Problem

Speaker: Ennes Kurt*

Date: 10.05.23

Contents: Introduce ultralimits/ ultrapowers of von Neumann algebras (including the hyperfinite II_1 factor), Connes Embedding Problem, hyperlinearity, and matrix-microstates/ random matrix formulation.

Sources: [8, §2.1-2.3, 3.6, 3.7], [19, §11.1, 11.5, 12.1-12.5], [17, §1], [21, §2.2], [7, §3].

7. Tensor Products of RF groups and RFD C^* -algebras.

Speaker: Akshara Prasad

Date: 17.05.23

Contents: Exactness and amenability for residually finite groups, residual finite dimensionality for $C^*(\mathbb{F}_2)$, $C^*(\mathbb{F}_2 \times \mathbb{F}_2) \cong C^*(\mathbb{F}_2) \otimes_{\max} C^*(\mathbb{F}_2)$, maximal tensor product of two RFD C^* -algebras is RFD iff there is a unique tensor norm, faithful traces on separable RFD C^* -algebras.

Sources: [5, §3.7 from pg.96], [6, Theorem 7, Corollary 9], [15, Proposition 3.19 (i)-(iii), Lemma 3.20], [19, §9.2], ([2, Prop 2.1, Lemma 3.2]).

8. Kirchberg's Conjectures

Speaker:

Date: 24.05.23

Contents: QWEP, permanence properties for (Q)WEP (direct products, conditional expectations, increasing nets, double duals, etc.) Kirchberg's QWEP conjecture, characterization via tensors on $C^*(\mathbb{F})$, Kirchberg's A conjectures

Sources: [5, §13.3 not including CEP], [15, Proposition 3.19], [14, §8], [19, §9.7].

9. Amenable Traces and between CEP and QWEP

Speaker:

Date: 07.06.23

Contents: Amenable (liftable) traces and WEP, proof that CEP is equivalent to QWEP.

Sources: [5, §6.2 and 13.3], [4, §3.1 and 6.3], (supplementary [3]), [19, §14.1], [15, §6].

10. Tsirleson's Problem

Speaker:

Date: 14.06.23

Contents: Quantum (Commuting) Correlation sets, universal C^* -algebras of projection valued measures and QWEP (going from $C^*(\mathbb{F})$ to $C^*(\mathbb{Z}_m^{**k})$).

Sources: [19, §13.1, Chapter 16], [13, §5], [21, §2.2], [17, §2].

11. Model Theory and CEP

Speaker: Andrea Vaccaro

Date: 21.06.23

Contents: $\text{CEP} \iff C^*(\mathbb{F})$ elementarily equivalent to a QWEP C^* -algebra (\iff computable universal theory for tracial type II_1 von Neumann algebras), with time other connections to embeddability questions in operator algebras.

Sources: [10], ([9, 11]).

12. $B(\mathcal{H})$ does not have LLP

Speaker:

Date: 28.06.23 (tentative)

Contents: Coding unitary matrices, Haagerup-Thorbjørnsen approach to bounding the coding value $C(k)$, RF property (T) approach, $B(\mathcal{H}) \otimes_{\min} B(\mathcal{H}) \neq B(\mathcal{H}) \otimes_{\max} B(\mathcal{H})$

Sources: [5, §13.5], [19, Chapter 18].

13. Group C^* -algebras without the LLP/WEP

Speaker:

Date: 05.07.23 (tentative)

Contents: Thom's non-residually finite hyperlinear property (T) group: Kirchberg's factorization property and residual finiteness under property (T); and/or $C^*(SL_3(\mathbb{Z}))$.

Sources: [5, §6.4], [1, §2.5, 4] [19, Chapter 17], [20, § 2], [12, §1, 2.2, 2.3, 3.1].

All sources should be available electronically and/or through the library. If you cannot access some of the source material, please let us know.

References

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